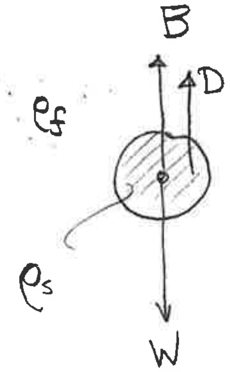
ASG v2
EX 11.3

(Terminal velocity problem)



a.) For terminal velocity, $B + D = W$

Where $B = \rho_f V_s g = \rho_f \frac{4}{3}\pi r^3 g$

$W = \rho_s V_s g = \rho_s \frac{4}{3}\pi r^3 g$

$D = \frac{1}{2}\rho_f C_D \pi r^2 v^2$ (for turbulent drag)

- Thus,

$$\underbrace{\rho_s \frac{4}{3}\cancel{\pi}\cancel{r^3}^{\,r} g}_{\text{weight}} = \underbrace{\rho_f \frac{4}{3}\cancel{\pi}\cancel{r^3}^{\,r} g}_{\text{buoyancy}} + \underbrace{\frac{1}{2}\rho_f C_D \cancel{\pi}\cancel{r^2} v_{term}^2}_{\text{drag}}$$

$$v_{term}^2 = \frac{\frac{4}{3}(\rho_s - \rho_f)\, g r}{\frac{1}{2}\rho_f C_D}$$

$$v_{term} = \sqrt{\frac{8 g r}{3 C_D}\left(\frac{\rho_s}{\rho_f} - 1\right)}$$ (for turbulent drag)

b.) If $r = 0.5\,mm = 5\times10^{-4}\,m$

$C_D \approx 0.5$ $\quad g = 9.8$ $\quad \nu = 1e\text{-}3$ Stokes (kinematic viscosity)

$\rho_s = 19.3\,g/cc$ $\quad \rho_f = 1.26\,g/cc$ for glycerol

So $v_{term} = 0.63\ m/s$

and the Reynolds number is $R = \frac{v_{term}\, r}{\nu} = \frac{(0.63)(5e\text{-}4)}{1e\text{-}3} = 0.315$

The Reynolds number is ≈ 0.3, which is in the range for laminar flow. So I guessed wrong, I should have used the formula for laminar drag.

Back to the drawing board, as they say.

$$D = 6\pi\mu r v$$

$$\rho_s \tfrac{4}{3}\cancel{\pi} r^{\cancel{3}\,2} g = \rho_f \tfrac{4}{3}\cancel{\pi} \cancel{r}^{\cancel{3}\,2} g + 6\cancel{\pi}\mu \cancel{r} v_{term}$$

$$v_{term} = \frac{\frac{4}{3}(\rho_s - \rho_f) g r^2}{6\mu}, \quad \mu = \nu \rho_f$$

$$v_{term} = \frac{2}{9}\frac{g r^2}{\nu}\left(\frac{\rho_s}{\rho_f} - 1\right)$$

$$v_{term} = \boxed{8 \text{ mm/sec}}$$

This gives $\boxed{R = 0.004}$, which is consistent with laminar flow.

c.) For the same ball falling through air, assume turbulent drag.

$$v_{term} = \boxed{\sqrt{\frac{8gr}{3C_D}\left(\frac{\rho_s}{\rho_f} - 1\right)}} = \boxed{20 \text{ m/s}}$$

$$R = \frac{20 \times 5e\text{-}4}{16e\text{-}6} = \boxed{625} \Rightarrow \text{turbulent (self-consistent)}$$