ASG v2 Ex 8.3 (Equations of motion for falling bodies)

a) $\int dt \left(v(t) \equiv \frac{dx}{dt} \right)$

$$\int dt\,(at + v_0) = \int dt \frac{dx}{dt}$$

$$\frac{1}{2}at^2 + v_0 t = \int dx$$

$$\frac{1}{2}at^2 + v_0 t = x - x_0$$

If $v_0 = x_0 = 0$, then $x = \frac{1}{2}at^2$

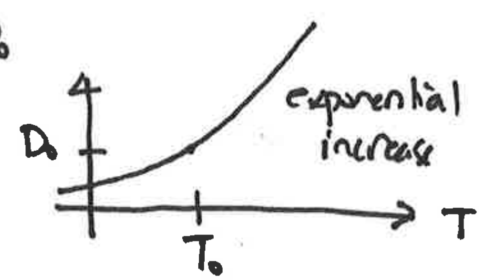
b) If $v \propto x$ then we can write $v = bx$

$$v \equiv \frac{dx}{dt}$$

$$\int_{T_0}^{T} dt = \int_{D_0}^{D} \frac{dx}{v}$$ ← write as definite integral

$$T - T_0 = \int_{D_0}^{D} \frac{dx}{bx} = \frac{1}{b} \ln x \Big]_{D_0}^{D}$$

$$T - T_0 = \frac{1}{b} \ln\left(\frac{D}{D_0}\right)$$



$$D_0 e^{b(T-T_0)} = D$$

$$D = D_0 e^{b(T-T_0)}$$ ← exponential increase in distance with time.

c) Salviati's criticism of Sagredo (i.e. that if the speed is proportional to distance fallen, then it implies instantaneous motion) isn't correct. Sagredo's ideas simply imply that the the time of fall is proportional to the natural log of D/D_o, where D_o is some constant value which is chosen to be the position at $T = T_o$.