$\phi_y = -\frac{\pi}{2}$, since $\epsilon = \phi_x - \phi_y = \frac{\pi}{2}$ for elliptically polarized waves. Thus

$$\boxed{\bar{E}_{iy} = 2E_0 \cos(\omega t - kz - \tfrac{\pi}{2})}$$

- This means that (using the same procedure as above for the reflected x-component):

$$\boxed{\bar{E}_{ry} = -2E_0 \cos(\omega t + kz - \tfrac{\pi}{2})}$$

b) Now we wish to find a complete, real vector expression for $\bar{E}(z,t)$ in the region $z<0$.

$$\bar{E}_{TOT} = \bar{E}_{ix} + \bar{E}_{rx} + \bar{E}_{iy} + \bar{E}_{ry}$$

$$= E_0 \Big\{ \cos(\omega t - kz)\,\hat{x} - \cos(\omega t + kz)\,\hat{x}$$

$$+ 2\cos(\omega t - kz - \tfrac{\pi}{2})\,\hat{y} - 2\cos(\omega t + kz - \tfrac{\pi}{2})\,\hat{y} \Big\}$$

- Using trig identities

$$\left\{ \begin{aligned} &\cos(A+B) = \cos A \cos B + \sin A \sin B \\ &\text{and } \cos(A-B) = \cos A \cos B - \sin A \sin B \\ &\text{so } \cos(A+B) - \cos(A-B) = 2 \sin A \sin B \end{aligned} \right\}$$

$$\cos(A-B) - \cos(A+B) = -2 \sin A \sin B$$