BD 5.5

a) $v_g = \frac{\partial \omega}{\partial k} \overset{?}{=} v - \lambda \frac{\partial v}{\partial \lambda}$

$= \frac{\partial \omega}{\partial \lambda} \frac{\partial \lambda}{\partial k}$

$\frac{\partial \omega}{\partial \lambda} = \frac{\partial}{\partial \lambda}\left(\frac{2\pi v}{\lambda} \right) = 2\pi \left(\frac{1}{\lambda} \frac{\partial v}{\partial \lambda} - \frac{v}{\lambda^2} \right) = k \frac{\partial v}{\partial \lambda} - \frac{k v}{\lambda}$

$\frac{\partial \lambda}{\partial k} = \frac{\partial}{\partial k}\left(\frac{2\pi}{k} \right) = \frac{-2\pi}{k^2} = \frac{-\lambda}{k}$

so $v_g = \frac{-\lambda}{k}\left(k \frac{\partial v}{\partial \lambda} - \frac{k v}{\lambda} \right) = -\lambda \frac{\partial v}{\partial \lambda} + v$

$$\boxed{v_g = v - \lambda \frac{\partial v}{\partial \lambda}}$$

b) $v_g = \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k}\left(\frac{ck}{n} \right) = c \frac{\partial}{\partial k}\left(\frac{k}{n} \right) = c \left(\frac{\partial \omega}{\partial k} \right) \frac{\partial}{\partial \omega}\left(\frac{k}{n} \right)$

$v_g \overset{1}{=} c\, v_g \frac{\partial}{\partial \omega}\left(\frac{k}{n} \right)$ (with v_g crossed out on both sides and replaced by 1)

$1 = \frac{c}{n} \frac{\partial k}{\partial \omega} + ck \frac{\partial}{\partial \omega}\left(\frac{1}{n} \right)$

$n = c/v_g - \frac{n c k}{n^2} \frac{\partial n}{\partial \omega}$ (with n and the exponent 2 crossed out)

$n + \frac{ck}{n} \frac{\partial n}{\partial \omega} = \frac{c}{v_g} \Rightarrow v_g = \frac{c}{n + \frac{ck}{n} \frac{\partial n}{\partial \omega}}$

$$\boxed{v_g = \frac{c}{n + \omega \frac{\partial n}{\partial \omega}}}$$