BB. 5.6

$\bar{E}_1 = E_0 \cos(5t + 10x)\hat{y}$ $\qquad x = \text{cm},\ t = \text{sec}$

$\bar{E}_2 = E_0 \cos(6t + 11x)\hat{y}$

a) $\bar{E} = E_0 \left(\cos(5t + 10x) + \cos(6t + 11x) \right)\hat{y}$ $\qquad \omega_1 = 5 \quad k_1 = -10$, $\quad \omega_2 = 6 \quad k_2 = -11$

$= 2E_0 \cos\left[\frac{1}{2}(5-6)t + \frac{1}{2}(-10+11)x\right] \cos\left[\frac{1}{2}(5+6)t + \frac{1}{2}(-10-11)x\right]\hat{y}$

$= 2E_0 \cos\left[-\frac{1}{2}t - \frac{1}{2}x\right] \cos\left[\frac{11}{2}t + \frac{21}{2}x\right]\hat{y}$

$= 2E_0 \cos\left[\frac{1}{2}t + \frac{1}{2}x\right] \cos\left[\frac{11}{2}t + \frac{21}{2}x\right]\hat{y}$

b) group velocity: $v_g = \frac{\partial\omega}{\partial k}$

There are two velocities: $v_1 = \frac{11/2}{21/2} = \frac{11}{21}\ \frac{\text{cm}}{\text{s}}$ in $-\hat{x}$ direction

$v_2 = \frac{1/2}{1/2} = 1\ \frac{\text{cm}}{\text{s}}$ in $-\hat{x}$ direction

The slower is the group velocity? $v_g = \frac{11}{21}$ cm/s

$v_g = \frac{\frac{1}{2}(\omega_1 - \omega_2)}{\frac{1}{2}(k_1 - k_2)} = \frac{5-6}{-10+11} = \frac{-1}{+1} = \boxed{-1\ \text{cm/s}}$

$v_p = \frac{\frac{1}{2}(\omega_1 + \omega_2)}{\frac{1}{2}(k_1 + k_2)} = \frac{11}{-21} = \boxed{\frac{-11}{21}\ \text{cm/s}}$

c) distance between points of zero amplitude?

$\Delta\omega = \omega_1 - \omega_2 = 1$ rad/s

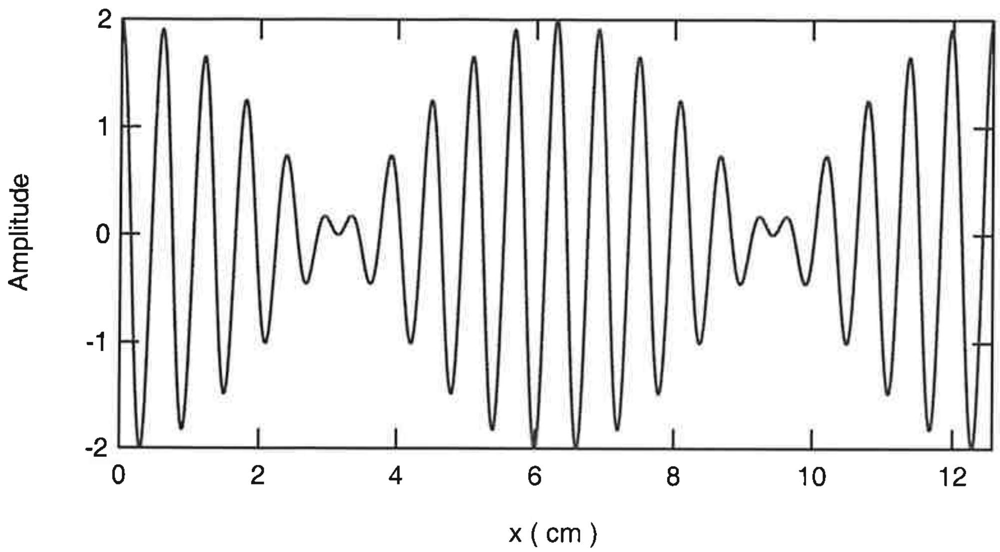
$\Rightarrow$ whole pattern repeats every $T = \frac{1}{f} = \frac{2\pi}{\Delta\omega} = 2\pi$ seconds

in this time, the envelope travels a distance 2π cm.

$v_p = \frac{\omega}{k}$ $\qquad v_g = \frac{\delta\omega}{\delta k}$

$k = \frac{2\pi}{\lambda}$ $\quad \lambda_1 = \frac{2\pi}{1}$

$\lambda_2 = \frac{2\pi}{21}$



Amplitude
2
1
0
-1
-2
0
2
4
6
8
10
12
x (cm)