Atkins 1.1   w is a function of x, y, and z.

a) I'll first prove that $\left(\frac{\partial w}{\partial x}\right)_{y,z} = 1\Big/\left(\frac{\partial x}{\partial w}\right)_{y,z}$

First, since $w = w(x,y,z)$ we can write

$$dw = \left(\frac{\partial w}{\partial x}\right)_{y,z} dx + \left(\frac{\partial w}{\partial y}\right)_{x,z} dy + \left(\frac{\partial w}{\partial z}\right)_{y,x} dz$$

Also, since $x = x(w,y,z)$

$$dx = \left(\frac{\partial x}{\partial w}\right)_{y,z} dw + \left(\frac{\partial x}{\partial y}\right)_{x,z} dy + \left(\frac{\partial x}{\partial z}\right)_{x,w} dz$$

Then, since

$$dx = \left(\frac{\partial x}{\partial w}\right)_{y,z}\left[\left(\frac{\partial w}{\partial x}\right)_{y,z} dx + \left(\frac{\partial w}{\partial y}\right)_{x,z} dy + \left(\frac{\partial w}{\partial z}\right)_{y,x} dz\right]$$
$$+ \left(\frac{\partial x}{\partial y}\right)_{x,z} dy + \left(\frac{\partial x}{\partial z}\right)_{y,w} dz$$

$$dx = \left(\frac{\partial x}{\partial w}\right)_{y,z}\left(\frac{\partial w}{\partial x}\right)_{y,z} dx + \left[\left(\frac{\partial x}{\partial w}\right)_{y,z}\left(\frac{\partial w}{\partial y}\right)_{x,z} + \left(\frac{\partial x}{\partial y}\right)_{x,z}\right] dy$$
$$+ \left[\left(\frac{\partial x}{\partial w}\right)_{y,z}\left(\frac{\partial w}{\partial z}\right)_{y,x} + \left(\frac{\partial x}{\partial z}\right)_{y,w}\right] dz$$

If we choose $dy = dz = 0$, which we can since they are independent variables, then $\left(\frac{\partial x}{\partial w}\right)_{y,z}\left(\frac{\partial w}{\partial x}\right)_{yz} = 1$

Or: $\boxed{\left(\frac{\partial w}{\partial x}\right)_{y,z} = 1\Big/\left(\frac{\partial x}{\partial w}\right)_{yz}}$ Q.E.D

b) now we'll prove that $\left(\frac{\partial w}{\partial x}\right)_{y,z}\left(\frac{\partial x}{\partial z}\right)_{w,y}\left(\frac{\partial z}{\partial w}\right)_{x,y} = -1$

If, in eq. ☺, we let $dx = dy = 0$,

then $\left(\frac{\partial x}{\partial w}\right)_{y,z}\left(\frac{\partial w}{\partial z}\right)_{y,x} = -\left(\frac{\partial x}{\partial z}\right)_{y,w}$

which gives us

$$\boxed{-1 = \left(\frac{\partial w}{\partial x}\right)_{y,z}\left(\frac{\partial x}{\partial z}\right)_{y,w}\left(\frac{\partial z}{\partial w}\right)_{y,x}}$$

Q.E.D.