Adkins 1.2 (This solution by Elise Sloey)

$\frac{A}{B} = C \Rightarrow \ln\left(\frac{A}{B}\right) = \ln C$ by taking ln of eqn.

$\ln A - \ln B = \ln C$ by log rules

$\left.\frac{\partial \ln A}{\partial x}\right)_y - \left.\frac{\partial \ln B}{\partial x}\right)_y = \left.\frac{\partial \ln C}{\partial x}\right)_y = \frac{1}{C}\left.\frac{\partial C}{\partial x}\right)_y$ taking $\left.\frac{\partial}{\partial x}\right)_y$

$\left.\frac{\partial \ln A}{\partial y}\right)_x - \left.\frac{\partial \ln B}{\partial y}\right)_x = \left.\frac{\partial \ln C}{\partial y}\right)_x = \frac{1}{C}\left.\frac{\partial C}{\partial y}\right)_x$ taking $\left.\frac{\partial}{\partial y}\right)_x$

$$\frac{\left.\frac{\partial \ln A}{\partial y}\right)_x - \left.\frac{\partial \ln B}{\partial y}\right)_x = \cancel{\frac{1}{C}}\left.\frac{\partial C}{\partial y}\right)_x}{\left.\frac{\partial \ln A}{\partial x}\right)_y - \left.\frac{\partial \ln B}{\partial x}\right)_y = \cancel{\frac{1}{C}}\left.\frac{\partial C}{\partial x}\right)_y}$$

dividing above 2 equations

$$\frac{\left.\frac{\partial C}{\partial y}\right)_x}{\left.\frac{\partial C}{\partial x}\right)_y} = \left.\frac{\partial C}{\partial y}\right)_x \left.\frac{\partial x}{\partial C}\right)_y = -\left.\frac{\partial x}{\partial y}\right)_C$$

reciprocal & reciprocity thm

$$\frac{\left.\frac{\partial \ln B}{\partial y}\right)_x - \left.\frac{\partial \ln A}{\partial y}\right)_x}{\left.\frac{\partial \ln A}{\partial x}\right)_y - \left.\frac{\partial \ln B}{\partial x}\right)_y} = \left.\frac{\partial x}{\partial y}\right)_C$$

proven