Atkins 2.2 (2 pts)

Take two identically constructed constant volume gas thermometers. Fill A with a "real" gas, which obeys

$$\left(P + \frac{a}{V^2}\right)(V-b) = RT,$$

where P is pressure, V is molar volume, R is the gas constant, T is temperature on an ideal gas (absolute) scale, and a and b are constants.

Fill thermometer B with an ideal gas, who obeys

$$PV = RT.$$

To show that, if both thermometers are calibrated at the ice and steam points, they will give identical values for temperature, we proceed as follows.

For a constant volume gas thermometer, the empirical temperature is given by

$$\Theta = 100 \left[\frac{P_\Theta - P_i}{P_s - P_i}\right] \quad (\star)$$

where P_Θ is the pressure at temperature Θ, P_i is the pressure at the ice point and P_s is the pressure

at the steam point.

For the gas in thermometer A,

$P = \frac{RT}{V-b} - \frac{a}{V^2}$. Substituting values of P_θ, P_i and P_s calculated from this formula into (*) gives

$$\theta_A = 100\left[\frac{\left(\frac{RT_\theta}{V-b} - \frac{a}{V^2}\right) - \left(\frac{RT_i}{V-b} - \frac{a}{V^2}\right)}{\left(\frac{RT_s}{V-b} - \frac{a}{V^2}\right) - \left(\frac{RT_i}{V-b} - \frac{a}{V^2}\right)}\right]$$

$$\theta_A = 100\left[\frac{T_\theta - T_i}{T_s - T_i}\right]$$

where θ_A is the reading on thermometer A for an arbitrary temperature T_θ on the ideal gas (absolute) scale.

Now, for the gas in thermometer B,

$P = \frac{RT}{V}$. Again, substituting values of P_θ, P_i & P_s calculated from this into (*) gives

$$\theta_B = 100 \left[\frac{\left(\frac{RT_o}{V}\right) - \left(\frac{RT_i}{V}\right)}{\left(\frac{RT_s}{V}\right) - \left(\frac{RT_i}{V}\right)} \right]$$

$$\theta_B = 100 \left[\frac{T_o - T_i}{T_s - T_i} \right]$$

This shows that $\theta_A = \theta_B$. In other words, the thermometers, once calibrated against the ice & steam points, must be in agreement, regardless of the medium filling them, for any other temperature.