Atkins 3.5 (3)

Find the work needed to compress a real gas obeying

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

from $V_1 = 5e\text{-}3\,m^3$ & $V_2 = 2e\text{-}5\,m^3$ at 300K, given $a = 8e4$ and $b = 3e\text{-}5$ and $N = 0.3$ mol.

$$W = \int_{V_1}^{V_2} p\,dV = N\int_{V_1}^{V_2} p\,dv \qquad \text{since } v = \frac{V}{N}$$

$$= N\int \left(\frac{RT}{v-b} - \frac{a}{v^2}\right) dv$$

$$= NRT\int \frac{dv}{v-b} - Na\int \frac{dv}{v^2}$$

$$= \left[NRT\ln(v-b) + Na/v\right]_{v_1}^{v_2}$$

$$\boxed{W = N\left[RT\ln\left(\frac{v_2-b}{v_1-b}\right) + a\left(\frac{1}{v_2} - \frac{1}{v_1}\right)\right]}$$

Now, plugging in $v_2 = 6.7e\text{-}5$ $\quad v_1 = 0.0167$

$$W = (0.3)\left[(8.31)(300)\ln\left(\frac{6.7e\text{-}5}{0.0167}\right) + 8e4\left(14925 - 200\right)\right]$$

$$= (0.3)\left[2493\ln(0.00222) + 8e4\,(14725)\right]$$

$$= 0.3\left[-15230 + 11.78\right] = \boxed{-4570 \text{ Joules}}$$