Atkins 3.8 (B)

Consider a 100 mg diamond at 77 K plunged into a bath of ^4_2He at 4.2 K. In the course of cooling to 4.2 K, 2.48e-5 m^3 (at 0°C and 1 atm) of Helium is evaporated. If the specific heat of diamond is $C = aT^3$, what is a?

Since we are dealing with C_p (constant pressure) the heat transfered from the diamond to the helium is just the change in enthalpy. Since

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p$$

and

$$\int dH = \int dQ_p = \int_{T_i}^{T_f} C_p dT$$

$$= +N \int_{T_i}^{T_f} aT^3 dT$$

$$-Q = \frac{N}{4} a T^4 \Big]_{T_i}^{T_f}$$

$$a = \frac{-4Q/N}{T_f^4 - T_i^4}$$

Now, how much heat was absorbed by the helium? Using $PV = NRT$ we can find the # of moles of helium, N.

$$N = \frac{PV}{RT} = \frac{(101{,}000\ \text{Pa})(2.48e\text{-}5\ \text{m}^3)}{(8.31)(273.15\ \text{K})}$$

$$= 0.0011\ \text{moles}$$

The latent heat of vaporization of helium is 21 kJ/kg,
The molar mass of helium is 4.003 kg/kmol

So we can calculate Q

$$Q = (0.0011\ \text{moles})(21\ \text{Joules/gram})(4.003\ \text{g/mol})$$

$$= 0.092\ \text{Joules}$$

Now we can find a.

$$a = \frac{0.092\text{J} / 0.0011\ \text{mol}}{77^4 - 4.2^4}$$

$$= \boxed{2.89e-12}$$