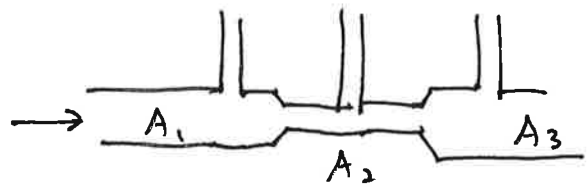
Adkins 3.9 (3)

For a flow rate of $1e-5 \frac{m^3}{s}$, what is the difference in height of water in the manometers below?



$A_1 = 100 mm^2 = A_3$

$A_2 = 50 mm^2$

Since whatever water flows into section 1 must also flow through 2 & 3, we have

$$A_1 v_1 = A_2 v_2 = A_3 v_3 \qquad (i)$$

where v_1, v_2 and v_3 are the speed of flow in the 3 sections. We also know, from (eq. 3.37) that the pressure and velocity in the sections are related.

$$P_1 V_1 + \frac{1}{2} v_1^2 = P_2 V_2 + \frac{1}{2} v_2^2 = P_3 V_3 + \frac{1}{2} v_3^2 \qquad (ii)$$

where P and V are the pressure and the volume per unit mass. We can now find the difference in pressure in sections 1 and 2. Since the volume per unit mass of water doesn't change ($V_1 = V_2 = V_3 = V$) we write.

$$P_1 V + \frac{1}{2} v_1^2 = P_2 V + \frac{1}{2} v_2^2$$

$$V(P_1 - P_2) = \frac{1}{2}(v_2^2 - v_1^2)$$

Using (eq. i) we write

$$P_1 - P_2 = \frac{1}{2V}\left(\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right)$$

$$= \frac{v_1^2}{2V}\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)$$

Now, the pressure is related to the fluid height by

$P = \rho g h$, where ρ is the mass

per unit volume, so

$$\cancel{\rho} g h_1 - \cancel{\rho} g h_2 = \frac{v_1^2}{2\cancel{V}}\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)$$

$$\boxed{h_1 - h_2 = \frac{v_1^2}{2g}\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)}$$

Now we need v_1. Since the flow rate $\frac{dV}{dt}$ is

$$\frac{dV}{dt} = A \cdot v$$

We can find

$$v_1 = \frac{dV}{dt} / A_1 = \frac{1e{-5}\, m^3/s}{100\ mm^2} \cdot \frac{(1000\ mm)^2}{1\ m^2} =$$

$$= 0.1\ m/s$$

So

$$h_1 - h_2 = \frac{(0.1)^2}{2(9.8)} \left(\left(\frac{100}{50}\right)^2 - 1 \right)$$

$0.0015 m = 1.5 mm$

(Manometer 2 is lower.)

There is no height difference between manometer 1 and 3.