TL 5-51

An electron & positron, each moving at $v = 3e6$ m/s (or $0.01c$), collide and annihilate, producing two photons.

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

a) First what is the wavelength of the incoming particles?
The electron & positron have the same mass, just opposite charges. So lets just compute the electron's λ. We will use eqn. 5-10

$$\frac{\lambda}{\lambda_c} = \frac{1}{\sqrt{2(E_k/E_0) + (E_k/E_0)^2}}$$

where λ_c = electron compton wavelength

E_k = kinetic energy of electron and

E_0 = rest energy of electron.

- Since the total energy of the electron is $E = E_k + E_0$

and since $E = \gamma m_0 c^2 = \gamma E_0$, where $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$

we see that $$\frac{\gamma E_0}{E_0} = \frac{E_k}{E_0} + \frac{E_0}{E_0}$$

or $$\frac{E_k}{E_0} = 1 - \gamma$$

Plugging in $\frac{v}{c} = 0.01$, we obtain $\frac{E_k}{E_0} = 0.00005$

- This means that the kinetic energy of the electron is miniscule compared to its rest energy (the energy associated with its rest mass.
- Plugging $\left(\frac{E_k}{E_0}\right)$ into 5-10 and using $\lambda_c = 2.426\,pm$, we get

$$\lambda_{electron} = 99.9988\,\lambda_c$$

$$\lambda_{electron} = 2.426 \times 10^{-10}\,m = \boxed{2.426\,\text{Å}}$$

b) Now, the photons that are produced will have energies that are equal to that of the annihilated electron (or positron). The energy of the electron is a fraction of a percent larger than its rest energy. So I'll just use the rest energy of the electron

$$E_0 = \boxed{511\,keV}$$

This will be the energy of each photon.

The photon momenta will each be $p = \frac{E}{c} = \boxed{2.73e\text{-}22\ kgm/s}$

And the photon wavelength will be $\lambda = \frac{h}{p} = 2.426e\text{-}12\,m$

$$\boxed{\lambda = 2.426\,pm}$$