

TL 7-66

Rigid body rotor has $KE = \frac{1}{2} I \omega^2$

$$KE = \frac{L^2}{2I}$$

For a quantum rotor, $E_l = \frac{l(l+1)\hbar^2}{2I}$, since $L^2 = \hbar^2 l(l+1)$

a) energy level diagram (in terms of $E_0 = \frac{\hbar^2}{2I}$)

E	l	$\frac{l(l+1)}{2I}$	E	$\frac{\Delta E}{E_0} = \frac{\Delta E}{E_0}$
E_5 —	5	30	$30E_0$	$10E_0 = 5E_1$
E_4 —	4	20	$20E_0$	$8E_0 = 4E_1$
E_3 —	3	12	$12E_0$	$6E_0 = 3E_1$
E_2 —	2	6	$6E_0$	$4E_0 = 2E_1$
E_1 —	1	2	$2E_0$	$E_0 = \frac{1}{2}E_1$
E_0 —	0	1	E_0	

b) If I take $E_1 = 2E_0$, I get the last column. This shows that the transitions between states with $\Delta l = \pm 1$ are integer multiples of $E_1 = \hbar^2/I$

c) For an H_2 molecule, $I = \frac{1}{2} m_p r^2 = 4.58 \times 10^{-48} \text{ kg m}^2$

$\left\{ \begin{array}{l} m_p = \\ r = 0.074 \text{ nm} \end{array} \right\}$

This means that $E_1 = \hbar^2/I = 2.13 \times 10^{-21} \text{ Joules}$

d) For the transition $l=1 \rightarrow l=0$

$$\frac{hc}{\lambda} = \frac{1}{2} E_1$$

$$\lambda = \frac{2hc}{E_1} = \boxed{169 \mu\text{m}}$$