TL 9-59 (Spectroscopy on H_2 molecule)

- Initially an H_2 molecule is in its ground electronic, vibrational & rotational state.
- It absorbs a photon of frequency $f = 1.356 \times 10^{14}$ Hz, which takes it to $v=1$, $\ell=1$ state in some electronic configuration.
- It then undergoes a vib-rot transition to $v=0$, $\ell=2$ state emitting a photon $f = 1.296e14$ Hz.

a) what is moment of inertia? First, some equations:

(9-14) $E_{or} = \frac{\hbar^2}{2I}$

(9-16) $I = \mu r_0^2$

(9-17) $\mu = \frac{m_1 m_2}{m_1 + m_2}$

(9-27) $f_{\ell \to \ell+1} = f + \frac{2(\ell+1)E_{or}}{h}$, $\ell = 0, 1, 2 \ldots$

(9-28) $f_{\ell \to \ell-1} = f - \frac{2\ell E_{or}}{h}$, $\ell = 1, 2, 3 \ldots$

I'll plug in our frequencies and ℓ-values and solve the equations...

$f_1 = 1.356 \times 10^{19} = f + \frac{2E_{or}}{h}$ first, absorption, event

$f_2 = 1.246 e19 = f - \frac{4E_{or}}{h}$ next, emission

- Solve these for f and for E_{or}

$$f = f_1 - \frac{2E_{or}}{h}$$

$$f_2 = f_1 - \frac{2E_{or}}{h} - \frac{4E_{or}}{h} = f_1 - \frac{6E_{or}}{h}$$

$$\frac{6E_{or}}{h} = f_1 - f_2 \Rightarrow E_{or} = \frac{h(f_1 - f_2)}{6} = 7.582 meV$$

- Now solve for I. $E_{or} = \frac{\hbar^2}{2I} \Rightarrow I = \frac{\hbar^2}{2E_{or}}$

$$\boxed{I = 4.577 \times 10^{-48} \ kgm^2}$$

b) Now I'll determine f and r_0.

$f = 131.9 THz = \boxed{1.319 \times 10^{14} Hz}$

Table 9-8 says $f = 1.32 \times 10^{14} Hz$. Very close. within 0.1%.

Since $I = \mu r_0^2$, $r_0 = \sqrt{\frac{I(m_1+m_2)}{m_1 m_2}} = \sqrt{\frac{2I}{m_H}}$

$$\boxed{r_0 = 0.07398 \ nm}$$

Table 9-8 says $r_0 = 0.074 nm$. nice.